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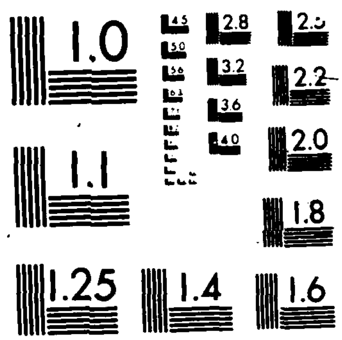
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A RANDOM ACCESS ALGORITHM FOR DIRECT SEQUENCE SPREAD-SPECTRUM PACKET RADIO NETWORKS.

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Department of

Electrical Engineering and Computer Science

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A Random Access Algorithm for
Direct Sequence Spread-Spectrum Packet
Radio Networks

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Abstract

We propose and analyze a limited sensing random access algorithm, named CRADS, for a multi-user and multi-receiver direct sequence spread spectrum system. Utilizing the regenerative character of the induced by the algorithm output process, we compute throughputs subject to an upper bound on the probability of erroneous data decoding; we also compute the expected per packet delays. The CRADS induces uniformly good delays within its stability region, and is particularly appropriate for environments where the users are highly mobile.

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1. Introduction

We consider packet radio multi-user spread-spectrum environments, where direct sequence spread spectrum techniques are deployed for protection against intelligent adversaries [1]. When the users in such environments are mobile and bursty, random access algorithms should be adopted, for efficiency in throughput and delay control. In this paper, we propose and analyze such an algorithm, named collision Resolution Algorithm for Direct Sequence (CRADS). The CRADS is a limited sensing random access algorithm utilizing receiver oriented direct sequence spread-spectrum patterns. In its design, the experience from random access algorithms for non spread-spectrum multi-user channels is utilized.

For the Poisson user model (large number of independent bursty users), and for transmissions through a single non spread spectrum channel with feedback, the existing stable random access algorithms belong to two distinct classes: The full sensing class, and the limited sensing class. The former requires that each user know the overall feedback history, and it includes algorithms such as those in [2], [3] and [4]. The latter requires that each user tune to the feedback broadcast only while he is blocked (i.e. from the time he generates a message to the time this message is successfully transmitted), and it includes algorithms in [5], [6], [7], [15], [16] and [17]. In mobile user environments, only the limited sensing class of random access algorithms is applicable, since the users can not then tune to the feedback broadcast whenever they move off the broadcast range.

Among others, Pursley [8] and Geraniotis and Pursley [9], studied the bit error probability induced in direct sequence spread spectrum multiple access channels, when transmitter oriented spread spectrum patterns are used. Pursley and Taipale [11] studied the packet error probability for spread spectrum packet radio with convolutional codes and Viterdi decoding; once more transmitter oriented spread spectrum patterns have been considered.

2. System Model

We consider the case where a large number of mobile independent bursty users use distinct per receiver direct sequence spread spectrum patterns, to transmit to a given

number, N_R, of semistatic receivers, through a common channel. We then model the overall user traffic as Poisson, and we also assume packet users and fixed length packets. In addition we consider a synchronous system, where the time of the common channel is divided into disjoint consecutive slots, and a packet transmission can only start at the beginning of some slot. A sufficient time-guard-band must be maintained between slots to guarantee packet synchronization in the face of differential delays due to the spatial distribution of users. Hence, the duration of a slot is at least equal to the packet length. We then measure time in slot units, where slot T occupies the time interval [T,T+1). We assume that each newly generated packet is destined for receiver k, $1 \leq k \leq N_R$, with probability $1/N_R$. Thus, if λ_o is the intensity of the overall Poisson user traffic, then the traffic per receiver is also Poisson, with intensity $\lambda = \lambda_o / N_R$. We also assume that the maximum delay with which some transmission from any user reaches any of the N_R receivers is α ($\alpha \ll 1$). Finally, we assume that the users, who access at a given slot any of the N_R receivers are uniformly spatially distributed around the receiver.

At the beginning of each slot, a distinct direct sequence spread spectrum pattern is assigned to each of the N_R receivers. Each such pattern is a periodic sequence of elements of $\{+1,-1\}$. The patterns belong to the class of maximal-length sequences (m-sequences) of period n, which are described in detail in section III of [10]. The pattern assigned to each receiver is called the signature sequence of the receiver.

Let us assume that a user wants to send his packet (i.e. a sequence of M bits $b = \{b_\ell\}_{\ell=0}^{M-1}$; $b_\ell \in \{+1,-1\}$) to a receiver of signature sequence $a = \{a_j\}_{j=0}^{n-1}$, at the beginning of slot T. For notational simplicity of the formulas to follow, we take T as the beginning of time (i.e. T=0). We then have that.

The binary data signal, which the user wants to transmit at slot T is of the form

$$b(t) = \sum_{\ell=0}^{M-1} b_\ell \, P_{T_b}(t - \ell T_b) \qquad (1)$$

where

$$P_{T_b}(t) = \begin{cases} 1 & 0 \leq t < T_b \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

and T_b is the data bit duration. The spectral-spreading signal is of the form

$$a(t) = \sum_{j=0}^{M\cdot n-1} a_j \, P_{T_c}(t-jTc) \qquad (3)$$

where

$$P_{T_c}(t) = \begin{cases} 1 & 0 \leq t \leq T_c \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

and T_c such that $T_b = n\ T_c$; hence exactly n pulses of the spectral spreading signal are contained in a data bit pulse. In (3) $a_{j+n_1} = a_j$, $0 \leq j \leq n-1$, where n_1 is a positive integer multiple of the period n of the signature sequence a.

The user sends to the receiver, at the beginning of slot T, the following signal

$$s(t) = A \cdot a(t) \cdot b(t) \cdot \cos(\omega_c t + \Theta) \qquad (5)$$

where ω_c is the carrier frequency and Θ is an arbitrary phase angle corresponding to the user, who sends the signal. We assume that Θ modulo-2π is uniformly distributed in the interfal $[0, 2\pi]$.

In general, in slot T, we may have I_1 users, each one with a packet for receiver 1, I_2 users each one with a packet for receiver 2,..., I_{N_R} users each one with a packet for receiver N_R. Let us denote by $a^k = \{a_j^{(k)}\}_{j=0}^{n-1}$, $1 \leq k \leq N_R$, the signature sequence of receiver k. Let us also denote by $b^i = \{b_\ell^{(i)}\}_{\ell=0}^{M-1}$, $1 \leq i \leq \sum_{k=1}^{N_R} I_k$, the data bit sequence of length M corresponding to the packet of user i, and by Θ^i , $1 \leq i \leq \sum_{k=1}^{N_R} I_k$, the phase angle corresponding to user i. Let us define

$$I_o = 0 \qquad (6)$$

$$I^k = \sum_{j=0}^{k-1} I_j \ , \quad 1 \leq k \leq N_R + 1 \qquad (7)$$

Then the signal transmitted at the beginning of slot T is of the form

$$s(t) = \sum_{k=1}^{N_R} \sum_{i=I^k+1}^{I^{k+1}} A\ a_k(t)\ b_i(t)\ \cos(\omega_c t + \Theta_i) \qquad (8)$$

where $a_k(t)$ and $b_i(t)$ are given by (1) and (3) respectively.

Let d_i^m , $1 \leq m \leq N_R$, $1 \leq i \leq I^{N_R+1}$, be the propagation delay between user i and receiver m. Then, the received signal at receiver m, $1 \leq m \leq N_R$ is

$$r_m(t) = \sum_{k=1}^{N_R} \sum_{i=I^k+1}^{I^{k+1}} A\, a_k(t-d_i^m)\, b_i(t-d_i^m) \cos(\omega_c(t-d_i^m) + \Theta_i) \qquad (9)$$

$$1 \leq m \leq N_R$$

If we denote by $\underline{d}^m \triangleq (d_1^m\ d_2^m \ldots d^m_{I^{N_R+1}})$, $1 \leq m \leq N_R$, and by $\underline{\Theta} \triangleq (\Theta_1\ \Theta_2 \ldots \Theta_{I^{N_R+1}})$ then, for every m $\underline{d}^m$ and $\underline{\Theta}$ are assumed to be independent random vectors, since they arise from unrelated physical phenomena. Furthermore we assume that for every m d_i^m and d_j^m are independent if $i \neq j$, and Θ_i and Θ_j are independent if $i \neq j$. Finally, it is reasonable to assume that the vectors b^i and b^j are independent if $i \neq j$, since they correspond to packets generated by different users.

For every m; $1 \leq m \leq N_R$, receiver m is a correlation receiver, which can lock on to at most one packet, generated by a user, who utilizes the signature sequence of receiver m. For example, in slot T receiver m can only lock on to at most one packet corresponding to one of the I_m users. Once the locking on procedure is accomplished, we assume that coherent demodulation of the signal is possible.

We will initially adopt the following assumption, which leads to N_R single-receiver multi-user decoupled systems, and which will be relaxed later in the paper.

<u>Assumption A1</u>. Simultaneous transmissions to different receivers do not interfer.

If assumption A1 is true, then the overall system consists of N_R identical and independent single receiver systems, each with input traffic Poisson with intensity λ. For each such system, we assume feedback broadcast capabilities. From then on, until assumption A1 is relaxed, the phrase "a receiver" will refer to any of the N_R receivers. The feedback information broadcasted by a receiver is ternary. In particular, a receiver distinguishes among the following events (i) The absence of a packet for a receiver within some slot is detected by the receiver, due to the low correlation between the incoming signal and the locally generated signature sequence corresponding to the

receiver. This outcome, E, is then broadcasted to the users. (ii) When a single packet is transmitted in a slot for a receiver, the receiver locks on to it. The locking on procedure is successful, because there is high correlation between the incoming signal and the locally generated signature sequence corresponding to the receiver. The receiver broadcasts then this success event, S. (iii) If at least two packets are simultaneously transmitted within a slot for a receiver, then depending on the corresponding delays with which these packets reach the receiver, either the receiver locks on to one packet or the locking on procedure fails. In the first case, the outcome S(success) is broadcasted by the receiver. In the second case, the event (collision) is broadcasted instead.

Let us elaborate more on case (iii). Let $K \geq 2$ packets be simultaneously transmitted within some slot for a receiver, and let them reach the receiver with delays, $d_1, d_2, \ldots, d_K$, where $d_1 \leq d_2 \leq \ldots \leq d_K \leq \alpha$. Now we differentiate:

(a) $d_j - d_1 \geq T_c$, $\forall\ j \neq 1$; then, the receiver locks on to the first arrived packet.

(b) $\exists\ j : d_j - d_1 < T_c$; Then, the locking on procedure fails, because the first and the second arrived packets are too close together for the receiver to distinguish. We now define

<u>Capture</u>. When, the locking on procedure for a receiver is successful we say that capture occurs; a slot where capture occurs is then a capture slot.

A receiver initiates the feedback broadcasts at the beginnings of slots, via direct sequence spread spectrum patterns that are orthogonal to those used for packet transmissions. Some synchronization information is included in the feedback broadcasts, so that all users, who try to access a receiver can synchronize their clocks with the clock of the receiver. All users, who try to access a receiver, should synchronize their clocks with the clock of the receiver, before attempting any transmission to the receiver. We assume a limited sensing environment, that is a user observes the feedback broadcasts from a receiver, from the moment a packet arrives at his buffer for the receiver until this packet is successfully transmitted. We now make two remarks.

<u>Remark 1</u>. The existence of clocks, for all users and receivers in the network, is

implied from our initial assumption of a slotted channel. The reason, for which we require that the users should accurately synchronize their clocks with the clock of the receiver, which they access, will be explained in the next section where the content of the feedback broadcast, S, is examined in detail.

Remark 2. We will make the assumption that the maximum propagation delay α between a user and a receiver, which the user accesses, equals the data bit duration T_b. Other values of α can be examimed as well (for example $\alpha=n'\cdot T_b$: n' integer, $n'\geq 2$) but they are not considered in this paper.

3. The Description of the CRADS Algorithm

The content of the feedback broadcast S will now be stated. A user, who transmits in some slot T, can compute the delay, d, between the time instant T, and the time when he receives from the addressed receiver the feedback corresponding to slot T. In the case of a single transmission within the slot T, the receiver can compute, d_1, that is the difference between the time when slot T begins, and the arrival time of the packet, by locking on to the packet; if $\ell T_c \leq d_1 < (\ell+1)T_c$; $0 \leq \ell \leq n-1$, the receiver broadcasts the integer ℓ in the place of S (success). In the case of multiple transmissions within the slot T, with corresponding delays $d_1 < d_2 \leq \ldots$, such that $d_j - d_1 \geq T_c$; $\forall\ j \geq 2$, the receiver can also compute, d_1, that is the difference between the time when slot T begins and the arrival time of the first arrived packet, by locking on to the first arrived packet; once more if $\ell T_c \leq d_1 < (\ell+1)T_c$; $0 \leq \ell \leq n-1$, the receiver broadcasts the integer ℓ in the place of S. Upon receiving the feedback broadcast ℓ (in the place of S) the users, who transmitted in slot T can identify their success by comparing the number ℓ with their own precomputed delays d (in particular to the number ℓ_1, such that $\ell_1 T_c \leq d < (\ell_1+1)T_c$; $0 \leq \ell_1 \leq n-1$) and the corresponding successfully transmitted packets depart the system. We note that in both the above cases (i.e single transmission in slot T, or multiple transmissions in slot T with delays $d_1 < d_2 \leq \ldots$, such that $d_j - d_1 \geq T_c$ $\forall\ j \neq 1$) there is only one packet with number ℓ_1 equal to the broadcasted integer ℓ ; so this packet leaves the system; the remaining transmitted packets (if any) can then identify their failure, by comparing ℓ to their own precomputed numbers ℓ_1.

It is easy to see that the successful implementation of the above procedure requires that the beginnings of slots must be an exact common reference for a receiver and the users, who access this receiver. We also observe that if a user transmits his packet in slot T the outcome of his transmission (C,S,E) will be available to him after slot T+1 begins. The description of CRADS will be facilitated if we make the zero-propagation delay assumption. This assumption implies that the outcome of slot T is available to all the users before slot T+1 begins. From now on, we make the zero-propagation delay assumption. This is not a restriction to our problem, because if the propagation delay is R slots ($R\geq 1$) we can treat the actual random access channel as R+1 - interleaved zero-propagation delay channels. Whatever random access algorithm is chosen, it is independently executed on each of the R+1 - interleaved channels. For more details see [3] pg. 121.

Subject to assumption A1 and the zero-propagation delay assumption, let us consider the single receiver system. Ler x_T denote the broadcast that corresponds to slot T, where x_T equals either E or C or S. The CRADS is then implemented by each user independently, as follows:

I. Each user initiates the algorithm at the time instant when he generates a new packet. He follows the rules of the algorithm until this packet is successfully transmitted, observing simultaneously the feedbacks broadcasted by the receiver. In the implementation of the algorithm the user uses a counter, whose value r_T at slot T is a nonnegative integer. The user transmits the packet in slot T, if and only if $r_T=0$. The values of the counter are updated as follows:

I.1 If the new packet is generated in $[T-1,T)$, then $r_T=0$

I.2 If $r_T=0$ and $x_T=S$, then,

I.2.a If the user identifies success for himself, then the packet is successfully transmitted and the algorithm stops.

I.2.b If the user identifies failure for himself, then he sets $r_{T+1}=0$.

I.3 If $r_T=0$ and $x_T=C$, then,

$$r_{T+1} = \begin{cases} 0 & ; \text{ w.p } 0.5 \\ 1 & ; \text{ w.p } 0.5 \end{cases}$$

I.4 If $r_T \geq 1$ and $x_T = S$, then $r_{T+1} = r_T$.

I.5 If $r_T \geq 1$ and $x_T = E$, then $r_{T+1} = r_T - 1$.

I.6 If $r_T \geq 1$ and $x_T = C$, then $r_{T+1} = r_T + 1$.

4. Analysis of the Algorithm

In this section, we will study the performance of the CRADS, subject to assumption A1 in section 2. The analysis is facilitated by the concept of a marker. The marker can be seen as an outside observer, who uses a counter. Denoting the value of this counter at slot T, R_T, then at time zero when the system operation begins, we set $R_o = 1$. After that, the values of R_T are updated as determined by rules I.4, I.5, and I.6 of the algorithm, in section 3, until the first time, T', that $R_{T'} = 0$. The time T' determines the end of the first session, as induced by the algorithm. Then, at T', $R_{T'}$ is set equal to one, the second session begins, and the above process is repeated. It can be easily seen that the session lengths are i.i.d random variables. Considering the single receiver system, let λ be the intensity of the Poisson traffic addressing the receiver, and let us define,

P_k : The probability of capture, given that k, $k \geq 1$, packets are transmitted within a single slot; the event of capture has been defined in section 2.

L_k : The expected length of a session, given that it starts with k packet transmissions.

L : The expected length of a session.

Assuming, as in section 2, that $\alpha = T_b = n \cdot T_c$ (n is the period of the signature sequence of the receiver), and considering highly mobile users, where in any slot time interval the users are randomly and uniformly spatially distributed, we conclude that the delays to the receiver in slot T, $\forall T$, are uniformly distributed in $[T, T+nT_c)$ and that they are independent for different slots. Then,

$$p_1 = 1$$

$$p_k = (1-n^{-1})^k \quad ; \quad k \geq 2 \qquad (10)$$

Also, we trivially conclude that

$$L = \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} L_k \tag{11}$$

Let us define, for p_k as in (10), and $U(x) = \begin{cases} 1 \ ; & x \geq 0 \\ 0 \ ; & x < 0 \end{cases}$

$$d_{1\ell} \triangleq e^{-\lambda} \frac{\lambda^{\ell}}{\ell!} \quad ; \ \ell \geq 0 \tag{12}$$

$$d_{k\ell} \triangleq 2^{-k+1} e^{-\lambda} (1-p_k) \sum_{j=0}^{\min(k,\ell)} \binom{k}{j} \frac{\lambda^{\ell-j}}{(\ell-j)!} + e^{-\lambda} p_k \frac{\lambda^{\ell+1-k}}{(\ell+1-k)!} U(\ell+1-k) \ ; \ k \geq 2, \ \ell \geq 0$$

Then from the description of the algorithm we easily derive that the expected lengths $\{L_k\}$ satisfy the following linear system

$$L_o = 1$$

$$L_k = \sum_{\ell=0}^{\infty} d_{k\ell} L_{\ell} + 1 \quad ; \ k \geq 1 \tag{13}$$

4.1 <u>System Stability</u>

In order to give a meaningful stability definition of our system, or in other words a definition of the throughput attained by CRADS, we need to examine two quantities i) The supremum of all Poisson intensities that provide a nonnegative and bounded solution for the linear system in (13); given n(the period of the signature sequence of the receiver) in $\alpha = T_b = n \cdot T_c$ we denote this quantity by λ_n^*. ii) The steady state probability with which, given a capture slot, the receiver decodes the captured packet incorrectly; we denote this probability p(E/C).

We will concentrate first on λ_n^*. Directly from the theory developed in [7], we now express the following theorem.

<u>Theorem 1</u>

(i) Given some finite positive integer N, let us consider the following truncated version of the system in (13).

$$x_o = 1$$

$$x_k = \sum_{\ell=0}^{N} d_{k\ell} x_{\ell} + 1 \quad ; \ 1 \leq k \leq N \tag{14}$$

Given n in $\alpha = n\cdot T_c$, let $\lambda_n^*(N)$ be the infimum of the Poisson intensities that do not give a nonnegative solution for the system in (14). Then $\lambda_n^*(N)$ is an upper bound to λ_n^*.

(ii) Given n in $\alpha = n\cdot T_c$, given N in (14), given $\lambda \le \lambda_n^*(N)$ let $\{x_k^*, 0\le k\le N\}$ be the nonnegative solution of the system in (14). Then, there exists $\lambda_n^o \le \lambda_n^*$, such that for every $\lambda \le \lambda_n^o$, there exist positive constants ε and a, a constant c, and a positive integer $N_o<N$, such that the system in (13) has a solution $y_k, k\ge 0$, which satisfies the following conditions:

$$
\begin{aligned}
&y_o = 1\ ,\ 0 \le y_k \le (1+\varepsilon)\,x_k^*\ ;\ 1\le k\le N_o \\
&0 \le y_k \le a\cdot k+c\ \ ;\ k>N_o
\end{aligned}
\tag{15}
$$

Selecting N=20 in (14), and following exactly the same methodology as in [7] we found the following bounds on λ_n^* for n=31

$$0.832 \le \lambda_{31}^* \le 0.835 \tag{16}$$

We will now concentrate on the quantity p(E/C). In order to compute this quantity, we need a theorem and a preliminary result.

<u>Theorem 2</u>

Let the discrete-time process $\{X_i\}_{i\ge 1}$ be regenerative with respect to the renewal sequence $\{R_i\}_{i\ge 1}$. Alto let $C_i = R_{i+1} - R_i$, i=1,2,... denote the length of the i-th regeneration cycle, and let f be a nonnegative, real valued, measurable function

If $C = E(C_1)<\infty$ and $S = E\{\sum_{i=1}^{C_1} f(X_i)\}<\infty$ then

$$\lim_{i\to\infty} \frac{1}{i}\sum_{j=1}^{i} f(X_j) = \lim_{i\to\infty} \frac{1}{i}\sum_{j=1}^{i} E(f(X_j)) = \frac{S}{C}\ ,\ \text{w.p.1}$$

Furthermore, if, in addition to the finiteness of C and S, the distribution of C_1 is not periodic, then $f(X_i)$ converges in distribution to a random variable $f(X_{ss})$ and

$$E(f(X_{ss})) = \frac{S}{C}$$

The above theorem is called the regeneration theorem and it will be used repeatedly. Now we proceed with the preliminary result.

Steady-State probabilities

Let $\{Z_k^{(i)}\}_{k\geq 0}$, i=0,1,... be such that if the time interval that corresponds to the kth slot contains i packets then $Z_k^{(i)} = 1$. Otherwise, $Z_k^{(i)} = 0$. For every given i, the process $\{Z_k^{(i)}\}_{k\geq 0}$ is regenerative with respect to the renewal sequence formed by the time instants when sessions begin. The regeneration cycle is then the length of a session. Given n and some λ in $(0,\lambda_n^*)$, let L be as in (11), and let $\Delta_k^{(i)}$ denote the expected number of slots within some session that starts with k packet transmissions, whose corresponding time intervals contain i packets. Then, for $d_{k\ell}$ as in (12), and for δ_{ij} being the Kronecker delta, we conclude from the operations of the algorithm that the numbers $\Delta_k^{(i)}$, satisfy the following linear system, for every i, and for λ in $(0,\lambda_n^*)$

$$\Delta_o^{(i)} = \delta_{io}$$

$$\Delta_k^{(i)} = \sum_{\ell=0}^{\infty} d_{k\ell}\, \Delta_\ell^{(i)} + \delta_{ik} \quad ; \; k\geq 1 \tag{17}$$

A theorem parallel to theorem 1 can be expressed for the system in (17), where we also have,

$$\Delta^{(i)} \triangleq \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} \Delta_k^{(i)} \tag{18}$$

Thus, we again conclude that $\Delta^{(i)}$ is bounded for every i and for λ in $(0,\lambda_n^*)$. Since L is also bounded, for all $\lambda\varepsilon(0,\lambda_n^*)$, the regeneration theorem applies, to give

$$\pi_i \triangleq \lim_{k\to\infty} P_r(Z_k^{(i)} = 1) = L^{-1}\, \Delta^{(i)} \quad ; \; i=0,1,2,... \tag{19}$$

Given i the quantity π_i in (19) is then the steady-state probability that a time interval corresponding to a channel slot, contains i packets. Upper and

lower bounds on the π_i are computed routinely via the method of truncated linear systems as in [7]. We computed those bounds for $i=0,1,2,3,4,5$ for $n=$ [illegible], and for various values of λ in the interval $(0,\lambda_n^*)$. Our results are included in table 1, where the bounds coincide to the corresponding digits in the table.

We are now in a position to define and compute p(E/C), or equivalently $p(CD/C) \overset{\Delta}{=} 1 - p(E/C)$ (i.e., the steady state probability with which, given a capture slot, the receiver decodes correctly the captured packet). In particular,

$$p(CD/C)=\left\{ \sum_{K=1}^{\infty} p(CD/C,K)\ p_K \quad \pi_K \right\} \left\{ p(C) \right\}^{-1} \tag{20}$$

where p_K ; $K\geq 1$ is given by (10), π_K ; $K\geq 1$ are the steady state probabilities defined in (19) and shown in Table 1, p(C) is the steady state probability that a slot is a capture slot, and p(CD/C,K) ; $K\geq 1$ is the probability with which, given a capture slot with K packets within, the receiver decodes correctly the captured packet; the expression for p(C) is the following

$$p(C) = \sum_{K=1}^{\infty} p_K \cdot \pi_K = \lambda \tag{21}$$

where the second equality in (21) can be proven by the regeneration theorem. From (20) and (21) it seems that in order to compute p(CD/C) we need to compute p(CD/C,K) ; $K\geq 1$.

<u>Computation of p(CD/C,K) ; K>1</u>

Let us examine the case of a capture slot with $K\geq 1$ packets within. Then, because of assumption A.1 and according to formula (9) the received signal at the receiver is of the form

$$r(t) = \sum_{k=1}^{K} A\ a_1\ (t-d_k)\ b_k(t-d_k)\ \cos(\omega_c(t-d_k) + \Theta_k) \tag{22}$$

In the case of multiple transmissions, $(K\geq 2)$, we assume that $d_1 < d_2 \leq \quad \leq d_K$ and $d_j - d_1 \geq T_c \quad \forall\, j \neq 1$ (since the slot is a capture slot), and taking under consideration that coherent demodulation of the first arrived packet is possible

then the output of the receiver during the ℓth bit of the first arrived packet is

$$Z_\ell = \int_{d_1+\ell T}^{d_1+(\ell+1)T} r(t)\, a_1(t-d_1) \cos(\omega_c(t-d_1) + \Theta_1)\, dt \quad ; \quad 0 \le \ell \le M-1 \tag{23}$$

The receiver decodes the ℓth bit correctly if ($b_{-1}^{(k)} \triangleq 0$; $2 \le k \le K$)

$$Z'_\ell \triangleq 1+T^{-1} \sum_{k=2}^{K} b_\ell^{(1)} [b_{\ell-1}^{(k)} R_{1,1}(d_k-d_1) + b_\ell^{(k)} \hat{R}_{1,1}(d_k-d_1)] \cdot \cos(\Theta_k-\Theta_1-\omega_c(d_k-d_1))>0$$

$$0 \le \ell \le M-1 \tag{24}$$

where $\hat{R}_{1,1}(\tau) = C_{1,1}(i) \cdot T_c + [C_{1,1}(i+1) - C_{1,1}(i)]\, (\tau-iT_c)$

$$; \ iT_c \le \tau < (i+1)\, T_c$$

$$; \ 0 \le i \le n-1 \tag{25}$$

and $R_{1,1}(\tau) = C_{1,1}(i-N)T_c + [C_{1,1}(i+1-N)-C_{1,1}(i-N)](\tau-iT_c)$

$$; \ iT_c \le \tau < (i+1)T_c \tag{26}$$

$$; \ 0 \le i \le n-1$$

$$\text{with } C_{1,1}(i) = \sum_{j=0}^{n-1-i} a_j^{(1)} \cdot a_{j+i}^{(1)} \qquad 0 \le i \le n-1 \tag{27}$$

$$\text{and } C_{1,1}(i) = \sum_{j=0}^{n-1+i} a_{j-i}^{(1)}\, a_j^{(1)} \qquad 1-n \le i < 0 \tag{28}$$

(for more details about (24), (25), (26), (27) and (28) see [8] pages 150, 151 and 165). A lower bound on p(CD/C,K) is the probability that all bits of the captured packet are decoded correctly. Hence,

$$p(CD/C,K) \ge p\left(\bigcap_{\ell=0}^{M-1} \{Z'_\ell > 0\} /C,K\right) \tag{29}$$

Equality in (29) is true only if the packet does not contain redundant bits for error correction. Furthermore, if

$$Z'_{min} \triangleq T^{-1} \sum_{k=2}^{K} [\,|R_{1,1}(d_k-d_1)| + |\hat{R}_{1,1}(d_k-d_1)|\,] \tag{30}$$

then obviously,

$$p\left(\bigcap_{\ell=0}^{M-1}\{Z'_\ell \geq 0\} \,/\, C,K\right) \geq p(Z'\mathrm{min} < 1 \,/\, C,K) \tag{31}$$

Note that $p(CD/C,1) = p(CD/C,2) = 1$. From now on we will concentrate on $p(Z'\mathrm{min} < 1 \,/\, C,K)$; $K \geq 3$.

After some manipulations we get that

$$p(Z'\mathrm{min} < 1 \,/\, C,K) =$$

$$= \int_0^{T-T_c} p\left(Z'\mathrm{min} < 1 \,/\, C,K,d_1=x,\left\{\bigcap_{k=2}^{K}\{d_k \geq d_1+T_c\}\right\}\right) \cdot K \left(\frac{T-T_c-x}{T^K}\right)^{K-1} dx \tag{32}$$

Given K, $d_1=x$ and $\{\bigcap_{k=2}^{K}\{d_k \geq d_1 + T_c\}\}$, $Z'\mathrm{min}$ is a sum of independent identically distributed random variables. The characteristic function corresponding to each term of the sum is given by the formula

$$f_x(\zeta) = \frac{1}{T-T_c-x}\int_{T_{c+x}}^{T} e^{i\zeta\{|R_{1,1}(\tau-x)|+|\hat{R}_{1,1}(\tau-x)|\}T^{-1}} d\tau \tag{33}$$

$$;0 \leq x < T-T_c$$

As a result the characteristic function of $Z'\mathrm{min}$, given K, $d_1=x$ and $\{\bigcap_{k=2}^{K}\{d_k \geq d_1+T_c\}\}$, is $[f_x(\zeta)]^{K-1}$.

Due to the above, we can write

$$p\left(Z'\mathrm{min} < 1 \,/\, C,K,\ d_1=x,\ \left\{\bigcap_{k=2}^{K}\{d_k \geq d_1+T_c\}\right\}\right) =$$

$$= \lim_{c\to\infty} \frac{1}{2\pi}\int_{-c}^{c} [f_x(\zeta)]^{K-1} \cdot \frac{1-e^{-i\zeta}}{i\zeta} d\zeta \tag{34}$$

From (32), (33) and (34) it turns out, that in order to compute $p(Z'\mathrm{min} < 1/C,K)$, we need to compute a double integral. Note, that once the signature sequence of the receiver is chosen, $f_x(\zeta)$ in (33) is known explicitly.

For the numerical results, we chose the signature sequence of the receiver to correspond to the first entry of Table A.1 (a) of [10]. This is the auto-optimal least-sidelobe-energy (AO/LSE) phase of a maximal length, sequence of period n=31. As far as the integral (34) is concerned, although it exists, it extends over an infinite interval. It would be desirable, if we could compute (34) over a finite

interval and estimate, at the same time, the error we commit. The form of $f_x(\zeta)$, which is completely determined by the signature sequence of the receiver, does not allow us to do so. This is the reason we resort to a trick. Instead of computing $p(Z'\min<1/C,K,d_1=x,\{\bigcap_{k=2}^{K}\{d_k\geq d_1+T_c\}\}$, we compute a lower bound of this probability. Hence, if we denote by Y a positive random variable uniformly distributed in [0,s] and independent from $Z'\min$ then

$$P(Z'\min<1/C,K,d_1=x,\{\bigcap_{k=2}^{K}\{d_k\geq d_1+T_c\}\}) \geq$$

$$P(Y+Z'\min<1/C,K,d_1=x,\{\bigcap_{k=2}^{K}\{d_k\geq d_1+T_c\}\} =$$

$$= \lim_{c\to\infty}\frac{1}{2\pi}\int_{-c}^{+c}[f_x(\zeta)]^{K-1}\ \frac{e^{is\zeta}-1}{\zeta s}\ \frac{1-e^{i\zeta}}{i\zeta}\ d\zeta \tag{35}$$

The absolute value of the integrand in (35) decreases as $0(\frac{1}{\zeta^2})$;, as a result we can compute the integral in (35) with arbitrary accuracy if we integrate over a proper finite interval. For the numerical results we took $s=10^{-2}$. Let us denote $p(Z'\min+Y<1/C,K)$ by $p^*(K)$; $K\geq 3$.

The values $p^*(K)$ for different values of K; $K\geq 3$ are given in Table 2. Note that

$$p(CD/C,K) \geq p^*(K)\ ;\ K\geq 3 \tag{36}$$

Let us define $p^*(1) = p(CD/C,1) = 1$, and $p^*(2) = p(CD/C,2) = 1$. We are now in a position to define and compute a lower bound of p(CD/C); we denote this lower bound $p_\ell(CD/C)$. So,

$$p_\ell(CD/C) = \sum_{K=1}^{\infty} p^*(K)\cdot p_K\cdot \pi_K \tag{37}$$

We can give now a meaningful stability definition of our system (i.e., throughput of CRADS)

Definition

Given n in $\alpha=nT_c$, given p* such that 0<p*<1, the throughput $\lambda_n^*(p^*)$ of CRADS is the maximum traffic intensity per receiver that maintains the value of the probability p(E/C) below than or equal to p^*.

We computed above a lower bound of p(CD/C) (namely $p_\ell(CD/C)$), or equivalently an upper bound on p(E/CD), which we denote $p_u(E/C)$; hence, given p^*, if we find the

maximum Poisson intensity such that $p_u(E/C) \leq p^*$ this will correspond to a lower bound on the throughput. For n=31 we find that

$$\lambda_n^*(p^*) \geq 0.8 \text{ if } p^*=10^{-3} \text{ and } \lambda_n^*(p^*) \geq 0.62 \text{ if } p^*=10^{-4} \quad (38)$$

4.2 Delay Analysis

Let the arriving packets be indexed, according to the order of their arrival time. Let $\mathcal{D}_j$ denote the delay of the jth packet; that is the time from its arrival to its successful transmission. Let $\mathcal{Q}_i$ denote the total number of packets that are successfully transmitted during the first i nonempty sessions. Then as in [7] and [15], we conclude that $\{\mathcal{Q}_i\}_{i\geq 0}$ is a renewal process, and that the process $\{\mathcal{D}_j\}_{j\geq 1}$ is regenerative with respect to $\{\mathcal{Q}_i\}_{i\geq 0}$, where the common regeneration cycle, $\mathcal{S}$, is the number of successfully transmitted packets during a nonempty session. Let us define

$$S \triangleq E(\mathcal{S}) \qquad D_S \triangleq E[\sum_{j=1}^{\mathcal{S}} \mathcal{D}_j] \quad (39)$$

Then from the regeneration theorem, we have that, if $\mathcal{S}$ is nonperiodic and if $S<\infty$ and $D_S<\infty$, then, $\mathcal{D}_j$ converges in distribution to a random variable $\mathcal{D}_\infty$, and, $D \triangleq \lim_{i\to\infty} i^{-1} \sum_{j=1}^{i} \mathcal{D}_j = \lim_{i\to\infty} i^{-1} E\{\sum_{j=1}^{i} \mathcal{D}_j\}$, w.p.1 while

$$D = E(\mathcal{D}_\infty) = D_S\, S^{-1}<\infty \quad (40)$$

From the operation of CRADS, we conclude that $P_r(\mathcal{S}=1) \neq 0$ so $\mathcal{S}$ is nonperiodic. Given n in $\alpha=nT_c$, and some λ in $(0,\lambda_n^*)$, we compute $S = (1-e^{-\lambda})\,\lambda\, L <\infty$, where L is as in (11). If we also show that $D_S<\infty$. for each λ in $(0,\lambda_n^*)$, then the regeneration theorem holds, and the parameter D in (40) is then the expected per packet delay induced by CRADS. Let $\mathcal{B}_j$ be the access delay of the ith packet (i.e. the time from its arrival until the beginning of the next slot); let $\mathcal{C}_j$ be the contention delay of the jth packet (i.e. the time from the beginning of the slot following its arrival until its successful transmission). Then $\mathcal{D}_j = \mathcal{B}_j + \mathcal{C}_j$, and

$$D_S = B + C \tag{41}$$

;where

$$B \triangleq E\{\sum_{j=1}^{S} B_j\},\ C \triangleq E\{\sum_{j=1}^{S} C_j\} \tag{42}$$

Given n and λ in $(0,\lambda_n^*)$, it can be easily found that $B = 2^{-1}S<\infty$, where S is as in (39). Thus, to show then that $D_S<\infty$, it suffices to prove that $C<\infty$, where C represents the expected cumulative per nonempty session contention delay, induced by the CRADS. Proceeding towards that direction, let us denote by $C_k, k\geq 0$, the expected cumulative per session contention delay, given that the session starts with k packet transmissions. Then for $\{L_k\}_{k\geq 0}$ as in section 3, for p_k as in (10) and for $d_{k\ell}$ as in (12) it is concluded from the operation of the algorithm, that $\{C_k\}_{k\geq 0}$ satisfies the following linear system

$$\begin{aligned} &C_o = 0 \\ &C_k = \sum_{\ell=0}^{\infty} d_{k\ell}\, C_\ell + f_k \end{aligned} \tag{43}$$

;where

$$\begin{aligned} &f_1 = 1 \\ &f_k = (1-p_k) \sum_{i=0}^{k} \sum_{j=0}^{\infty} \binom{k}{i} 2^{-k} e^{-\lambda} \frac{\lambda^j}{j!} (k-i)\, L_{i+j} + k\ ;\ k\geq 2 \end{aligned} \tag{44}$$

Upper and lower bounds of the quantities $\{C_k\}_{k\geq 1}$ in (43) are computed via methods as in theorem 1, and in [7]. Given n, for N_o and x_k^* as in (15), and for λ in $(0,\lambda_n^*)$, we find as in [7] that instead of the inequalities in (15), we now have

$$\begin{aligned} &0 \leq C_k \leq db(1+\varepsilon)\, x_k^* + dk^2\ ;\ 1\leq k\leq N_o \\ &0 \leq C_k \leq db(ak+c) + dk^2\ ;\ k>N_o \end{aligned} \tag{45}$$

;where ε,a,c are as in (15), and where d and b are positive and bounded constants. Due to (45) and the equation

$$C = (1-e^{-\lambda})^{-1} \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} C_k$$

We conclude that given n and λ_n^*, C is bounded. Thus D_S in (41) is then bounded as

well, and we can compute the expected per packet delay as follows

$$D = D_S S^{-1} = [B+C]\ [(1-e^{-\lambda})^{-1}\lambda L]^{-1} = [2^{-1}S+C]\ (1-e^{-\lambda})\lambda^{-1}L^{-1} =$$

$$= 0.5+\lambda^{-1}[\sum_{k=0}^{\infty} e^{-\lambda}\frac{\lambda^k}{k!}L_k]^{-1}\cdot[\sum_{\ell=0}^{\infty} e^{-\lambda}\frac{\lambda^\ell}{\ell!}C_\ell] \quad (46)$$

Upper and lower bounds, D_u and D_ℓ, on D are computed as in [7], via the method of truncated linear systems [13]. The specifics of those truncations are routine and are omitted here. The computed bounds D_u and D_ℓ are included in Table 3 for n=31 in $\alpha=nT_c$, and are identical to the digits shown in the table. In figure 1, D is plotted against λ.

5. Relaxation of Assumption A1 - Multiple Receiver Model

To this point, we assumed lack of interference from transmissions addressing different receivers, which allowed the isolation of each single receiver model. In this section, we will relax this assumption. In particular, we will consider two receivers. Receiver 1 is the receiver of the previous section with signature sequence a_1 of period n and receiver 2 is another receiver with signature sequence a_2 of the same period. In the previous section, we computed a lower bound on the throughput attained by CRADS, when packet transmissions directed to receiver 1, are not affected by packet transmissions directed to other receivers. In this section, we will compute a lower bound on the throughput attained by CRADS when packet transmissions directed to receiver 1 are affected by packet transmissions directed to receiver 2. As we will see, more general situations, when packet transmissions to receiver 1 are affected by packet transmissions to the remaining N_R-1 receivers can be treated similarly, but they are not considered here.

The feedback model described in section 2 is valid for both receivers. Hence, in the two-receiver model, we have a CRADS algorithm operating for the packets addressed to receiver 1, and another CRADS algorithm operating for the packets addressed to receiver 2. It is easy to see, that these two algorithms operate

concurrently and indendently from each other. As a result λ_n^*, the steady state probabilities, and the delay analysis of the single receiver model are still valid for each CRADS mentioned above. The only difference between the single and the two-receiver model is that the value of the quantity p(E/C) (defined in the previous section) is now changed. Let us concentrate on receiver 1. Now

$$p(CD/C) = \left\{ \sum_{K_1=1}^{\infty} \sum_{K_2=0}^{\infty} p(CD/C,K_1,K_2)\, P_{K_1} \cdot \pi_{K_1} \cdot \pi_{K_2} \right\} \left\{ p(C) \right\}^{-1} \tag{47}$$

where K_1 is the number of packets addressed for receiver 1 and K_2 the number of packets addressed for receiver 2. Furthermore

$$P(C) = \sum_{K_1=1}^{\infty} \sum_{K_2=0}^{\infty} P_{K_1} \cdot \pi_{K_1},\ \pi_{k_2} = \lambda \tag{48}$$

The received signal at receiver 1, in a slot with K_1 packets addressed for receiver 1 and K_2 packets addressed for receiver 2, is of the form (see also formula 9)

$$\begin{aligned} r(t) = & \sum_{k=1}^{K_1} A\, a_1(t-d_k)\, b_k(t-d_k) \cos(\omega_c(t-d_k) + \Theta_k) + \\ & + \sum_{k=1}^{K_2} A\, a_2(t-d_k')\, b_k'(t-d_k') \cos(\omega_c(t-d_k') + \Theta_k') \end{aligned} \tag{49}$$

If the above slot is a capture slot for receiver 1 (assume $d_1 < d_2 \leq \dots \leq d_K$ that $d_j - d_1 \geq T_c\ \forall_j \neq 1$) then, the output of receiver 1 during the ℓth bit of the first arrived packet for the receiver is

$$Z_\ell = \int_{d_1+\ell T}^{d_1+(\ell+1)T} r(t) a_1(t-d_1) \cos(\omega_c(t-d_1) + \Theta_1) dt \quad ; 0 \leq \ell \leq M-1 \tag{50}$$

Receiver 1 decodes the ℓth bit correctly if $(b_{-1}^{(k)} \triangleq 0;\ 2 \leq k \leq K_1,\ b'^{(k)}_{-1} = 0;\ 2 \leq k \leq K_2)$

$$\begin{aligned} Z_\ell = 1+T^{-1} \cdot \Bigg\{ & \sum_{k=1}^{K_1} b_\ell^{(1)} \cdot [b_{\ell-1}^{(k)} R_{1,1}(d_k-d_1) + b_\ell^{(k)} \hat{R}_{1,1}(d_k-d_1)] \cos(\Theta_k - \Theta_1 - \omega_c(d_k-d_1)) + \\ & + \sum_{k=1}^{K_2} b_\ell^{(1)} \cdot \cos[\Theta_k' - \Theta_1 - \omega_c(d_k'-d_1)] \cdot \{ [b'^{(k)}_{\ell-1} R_{2,1}(d_k'-d_1) + b'^{(k)}_\ell \hat{R}_{2,1}(d_k'-d_1)] \cdot I_{d_k' > d_1} + \\ & + [b'^{(k)}_{\ell+1} R_{1,2}(d_1-d_k') + b'^{(k)}_\ell \hat{R}_{1,2}(d_1-d_k')]\, I_{d_k' < d_1} \Bigg\} > 0 \end{aligned} \tag{51}$$

where I_A is the indicator function of the event A, and

$$\hat{R}_{2,1}(\tau) = C_{2,1}(i)T_c + [C_{2,1}(i+1) - C_{2,1}(i)]\,(\tau - iT_c) \tag{52}$$
$$;iT_c \le \tau < (i+1)T_c$$
$$;0 \le i \le n-1$$

$$R_{2,1}(\tau) = C_{2,1}(i-N)\,T_c + [C_{2,1}(i+1-N) - C_{2,1}(i-N)](\tau - iT_c) \tag{53}$$
$$;iT_c \le \tau < (i+1)T_c$$
$$;0 \le i \le n-1$$

$$C_{2,1}(i) = \sum_{j=0}^{n-1-i} a_j^{(2)}\,a_{j+i}^{(1)} \tag{54}$$
$$;0 \le i \le n-1$$

$$C_{2,1}(i) = \sum_{j=0}^{n-1+i} a_{j-i}^{(2)}\,a_j^{(1)} \tag{56}$$
$$;1-n \le i < 0$$

$\hat{R}_{1,2}(\tau)$, $R_{1,2}(\tau)$, $C_{1,2}(i)$, are defined as in (52), (53), (54) and (55) with the roles of subscripts and superscripts 1 and 2 interchanged.

Similarly as in section 3 we find that

$$p(CD/C,K_1,K_2) \ge p(Z'\min < 1 \,/\, C,K_1,\,K_2) \tag{57}$$

where now

$$Z'\min = T^{-1}\sum_{k=1}^{K_1}\left[\,|R_{1,1}(d_k - d_1)| + |\hat{R}_{1,1}(d_k - d_1)|\,\right] +$$

$$+\,T^{-1}\sum_{k=1}^{K_2}\left(|R_{2,1}(d_k' - d_1)| + |\hat{R}_{2,1}(d_k' - d_1)|\right) I_{d_k' \ge d_1} + \left(|R_{1,2}(d_1 - d_k')| + |\hat{R}_{1,2}(d_1 - d_k')|\right) I_{d_k' < d_1} \tag{58}$$

Following the same procedure as in section 3 we compute a lower bound on the probability $p(CD/C,K_1,K_2)$ which we denote $p^*(K_1,K_2)$. In our computations the signature sequence of receiver 1 is the first entry of Table A.1(a) of [10] and the signature sequence of receiver 2 is the second entry of Table 4.1(a) of [10]. If the above signature sequences are used for receiver 1 and receiver 2 it is easy to see (from (51)) that $p(CD/C,1,1) = p(CD/C,1,2) = 1$. Thus $p^*(K_1,K_2)$ is computed for all other values of

K_1 and K_2; note also that $p^*(K_1,0) = p^*(K_1)$. In Table 4 the values of $p^*(K_1,K_2)$ for different values of K_1, K_2 are included. As in the single receiver model a lower bound on p(CD/C), which we denote $p_\ell(CD/C)$, can now be defined and computed; for uniformity of notation we define $p^*(1,1) = p(CD/C,1,1) = 1$ and $p^*(1,2) = $ $= p(CD/C,1,2) = 1$. So

$$p_\ell(CD/C) = \sum_{K_1=1}^{\infty} \sum_{K_2=0}^{\infty} p^*(K_1,K_2)\, p_{K_1} \cdot \pi_{K_1} \pi_{K_2} \tag{59}$$

We computed a lower bound of p(CD/C) (namely $p_\ell(CD/C)$) or equivalently an upper bound on p(E/CD), which we denote $p_u(E/C)$; hence, given $p^*(0<p^*\leq 1)$, if we find the maximum Poisson intensity such that $p_u(E/C) \leq p^*$ this will correspond to a lower bound on the throughput of CRADS. For n=31 and the two-receiver model.

$$\lambda_n^*(p^*) \geq 0.46 \text{ if } p^*=10^{-3} \text{ and } \lambda_n^*(p^*) \geq 0.33 \text{ if } p^*=10^{-4} \tag{60}$$

6. Comments and Conclusions

In this paper we presented and analyzed a limited sensing random access algorithm, for direct sequence multi-user and multi-receiver spread spectrum systems with slotted transmission channel. We considered distinct signature sequence per receiver, which belong to the class of maximal length sequences of period n=31; maximal length sequences of period n=63, 127, 255 can be considered as well. The maximum number of maximal length sequences of period n is rather limited (see [10]). Furthermore, although maximal length sequences have good autocorrelation properties, it is not always possible to find enough m-sequences with good crosscorrelation properties. If it is necessary to obtain a larger class of sequences of period n which have good crosscorrelation properties (at the expense of slightly worse autocorrelation properties) sequences other than the m-sequences, must be considered. One such class of sequences are the Gold sequences; for more details about Gold sequences see [10].

We considered a Poisson user model. This model best reflects environments

where users are highly mobile. In addition random access algorithms devised for such a model are robust in the presence of changing traffic. For traffic changes within their stability region, they remain stable, and they induce uniformly good delays. The algorithm in this paper is a limited sensing random access algorithm. That is, each user is required to monitor the feedback broadcast, only while he is blocked. This property is generally very attractive, and is indispensable in environments where the users are highly mobile.

Concluding, we point out that in the construction of our models, papers [8], and [18], were very helpful.

	λ	π_o	π_1	π_2	π_3	π_4	π_5
$n = 31$	0.1	0.89964001	0.94801503D-01	0.53302816D-02	0.22021524D-03	0.77313057D-05	0.24835363D-06
	0.2	0.79820591	0.17762248	0.21759949D-01	0.20318262D-02	0.16620615D-03	0.12646024D-04
	0.3	0.69598755	0.24574664	0.49258544D-01	0.77545852D-02	0.10893924D-02	0.14308678D-03
	0.4	0.59191098	0.29590885	0.86493332D-01	0.20335602D-01	0.43070839D-02	0.85152170D-03
	0.5	0.48540596	0.32411613	0.13023442	0.42842939D-01	0.12716168D-01	0.34988248D-02
	0.6	0.37513902	0.32530898	0.17459672	0.77479087D-01	0.30788505D-01	0.11209475D-01
	0.7	0.25865140	0.29262555	0.21002504	0.12402223	0.64273016D-01	0.29897467D-01
	0.8	0.13112265	0.21562421	0.22162857	0.17764270	0.11894798	0.69178453D-01

Table 1

Steady State Probabilities

Table 2

K	$p^*(K)$
3	0.906266
4	0.876813
5	0.847950
6	0.819403
7	0.790614
8	0.758138
9	0.710626
10	0.631407

Table 3

Expected Delays; $D=D_u=D_\ell$ in the values included.

λ	D for n=31
0.1	1.6356033
0.2	1.8479264
0.3	2.1928313
0.4	2.7904166
0.5	3.9373853
0.6	6.5333339
0.7	14.508641
0.8	80.084165

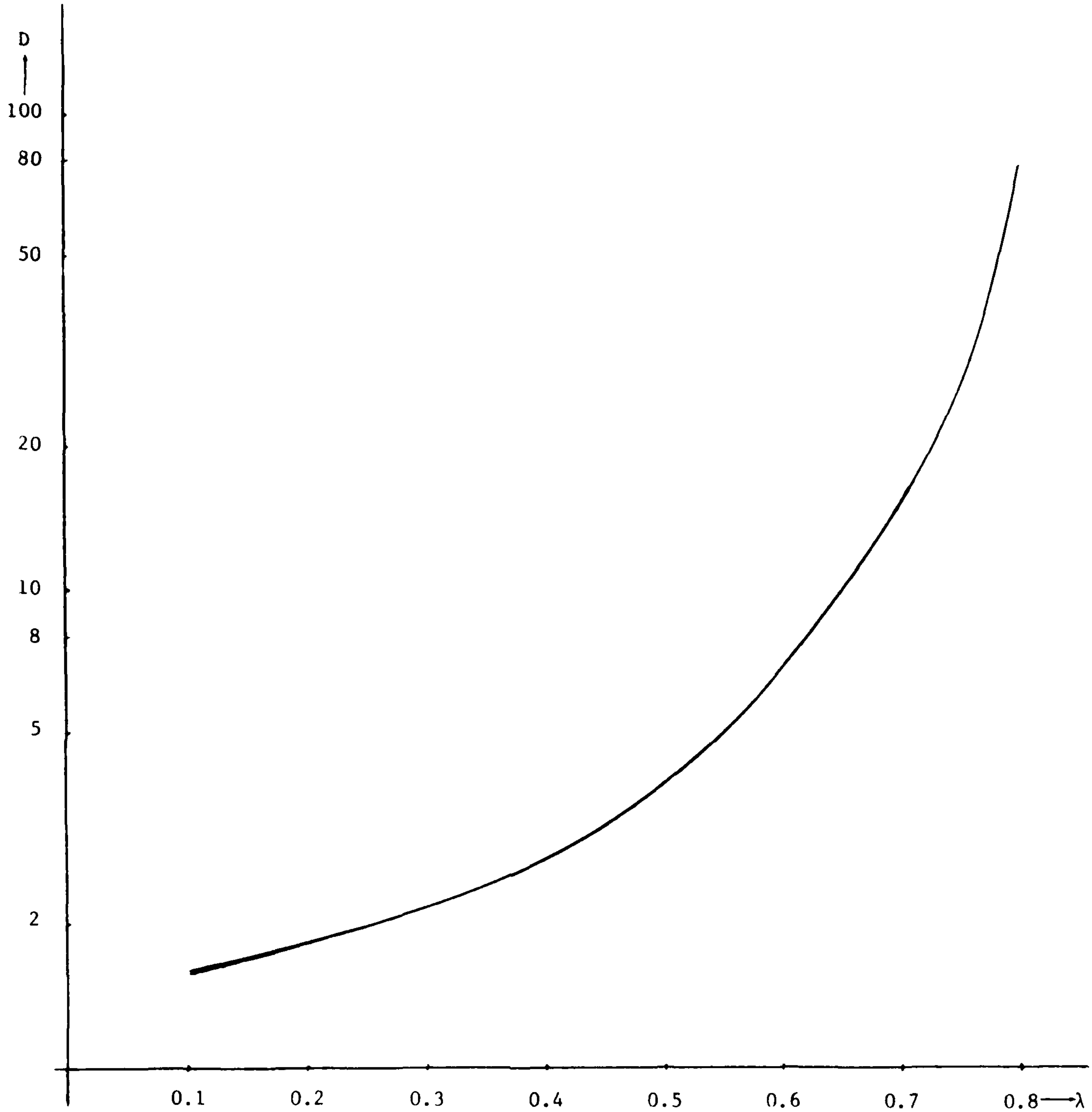

Figure 1

Expected Delays

Table 4

K_1	K_2	$p^*(K_1,K_2)$
1	3	0.999998
1	4	0.995308
1	5	0.903113
1	6	0.637476
1	7	0.332442
1	8	0.130947
1	9	0.040406
2	1	0.936479
2	2	0.936391
2	3	0.935816
2	4	0.903877
2	5	0.724831
2	6	0.431092
2	7	0.189740
2	8	0.064031
3	1	0.906085
3	2	0.905731
3	3	0.897487
3	4	0.797254
3	5	0.540592
3	6	0.267319
3	7	0.099096
4	1	0.876304
4	2	0.874336
4	3	0.832138
4	4	0.642273
4	5	0.361050
4	6	0.148724
5	1	0.846789
5	2	0.833282
5	3	0.720073
5	4	0.464660
5	5	0.215641
6	1	0.815151
6	2	0.762451
6	3	0.565387
6	4	0.299882
7	1	0.770891
7	2	0.646648
7	3	0.396674
8	1	0.695332
8	2	0.494761
9	1	0.577772

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